Effects of Asymmetry in Landfill Gas Flow

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Landfill waste disposal

Responsible waste disposal is necessary for the health of the people, the environment, and for aesthetic reasons. The most common method of waste disposal in North America is in landfills. Landfill gas refers to the hazardous emissions produced by the waste in a landfill. The gas is controlled by extraction via wells placed horizontally or vertically, and subsequent removal from the area. Predicting the effect of a change in the function of a single well on the other wells in the landfill is very important to the design and operation of a landfill. Such predictions cannot be accomplished by simple field measurements, as they quickly lose usefulness over time as the waste degrades.

Method of analysis

The analysis begins with a basic unit of two cells, positioned side by side or stacked vertically. The total (integrated) normal flux across the centre line equals the difference in production of the two wells. The integrated normal flux between cells is one of very few values that is easy to access in the field with a high certainty, and thus traceable over time. This study charts the dependence of the flux on the relative suction strength of the two wells under general operating conditions and then extends the results to arrays of more than two cells. The configurations shown in figures 1 and 2 are horizontal 1×2 and vertical 2×1 respectively. Aspect ratios in the range $1 \leqslant \ell_x\big/\ell_y \leqslant 3$ were tested with the cell depth ℓ_y held fixed. The bottom and sides of the landfill were set to have no flow normal to the boundary. The top was tested with two different conditions: zero normal flux, and a fixed atmospheric or slightly sub-atmospheric pressure. The pipe boundaries were set to their induced suction values.

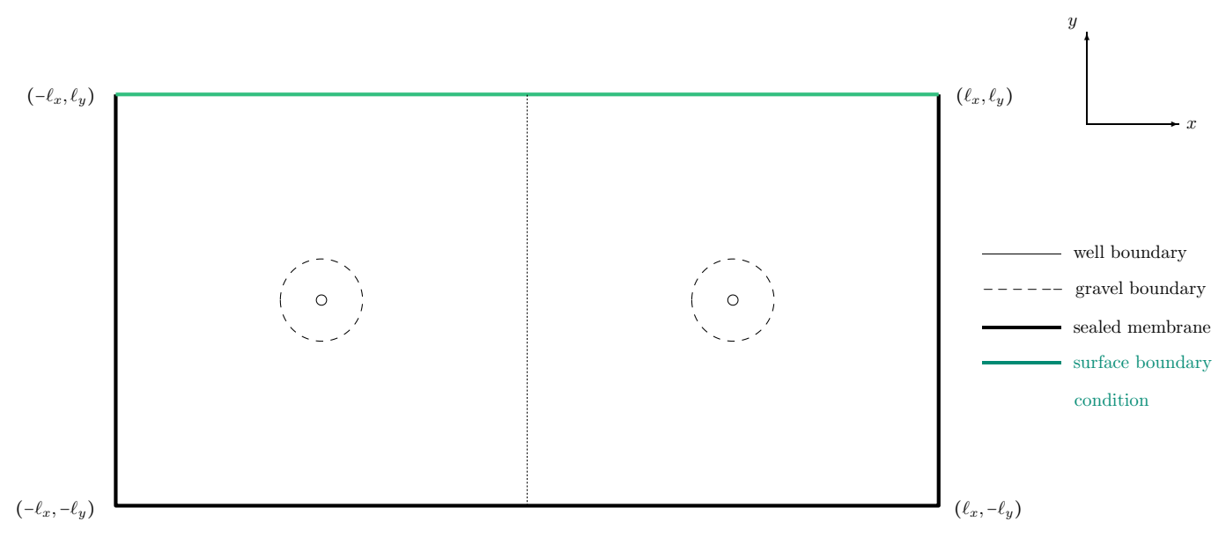

Figure 1 Solution domain for the horizontal 1×2 configuration (dimensions not to scale).

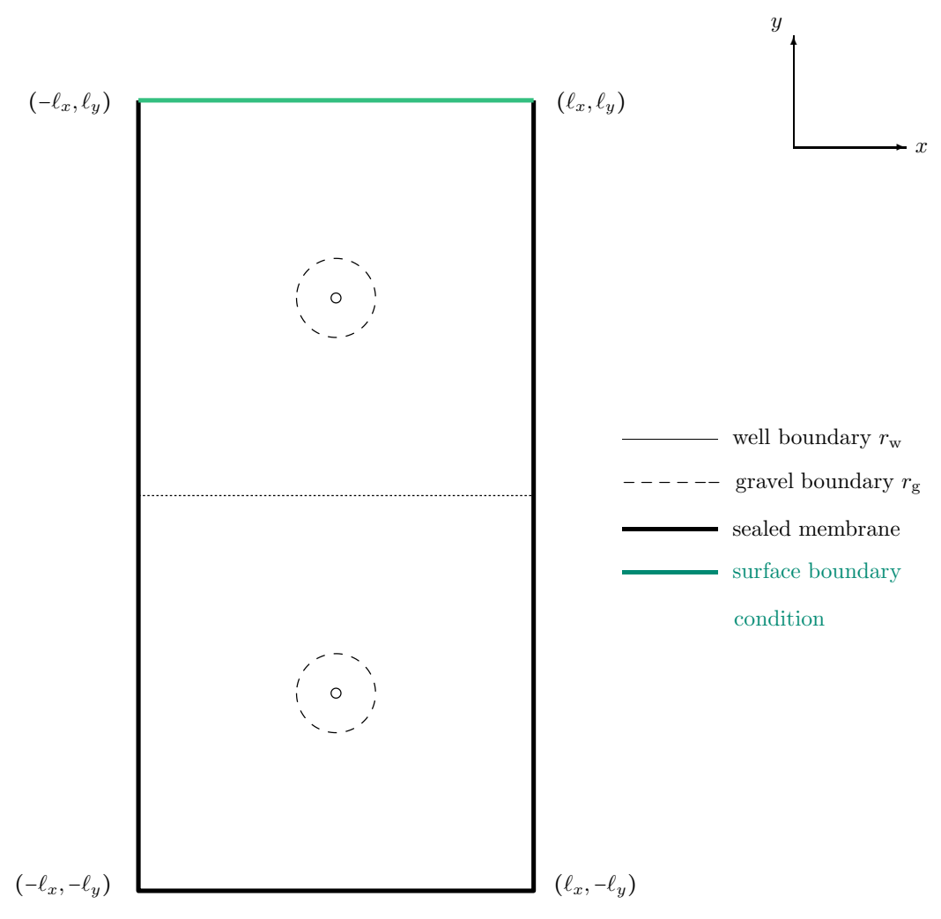

Figure 2 Solution domain for the vertical 2×1 configuration (dimensions not to scale).

Methodology for different geometries

♣ Horizontal 1×2: right well assigned a nominal value $p_{\mathrm{w_r}}$, in the range $-3.75 \leqslant p_{\mathrm{w_r}} \leqslant -1.25$kPa; left well varied between $p_{\mathrm{w_r}}$ (fully functional) and p_{atm} (dysfunctional) by:

$$p_{\mathrm{w_l}} = p_{\mathrm{w_r}} + \left(p_{\mathrm{atm}} - p_{\mathrm{w_r}}\right)f_p, \qquad 0 \leqslant f_p \leqslant 1.$$

$f_p = 0$: equal suction at both wells. $f_p > 0$: the right pipe dominates, collecting more gas than the left. $f_p = 1$: left pipe entirely dysfunctional.

♣ Vertical 2×1: asymmetry for the fixed surface pressure condition with no gravity and in both pressure conditions with gravity leads to both the top and bottom pipes needing to be tested separately. Therefore, f_p might be positive (top pipe dominant $p_{\mathrm{w_t}} < p_{\mathrm{w_b}}$) or negative (bottom pipe dominant $p_{\mathrm{w_b}} < p_{\mathrm{w_t}}$) in:

$$p_{\mathrm{w_b}} = p_{\mathrm{w_t}} + \left(p_{\mathrm{atm}} - p_{\mathrm{w_t}}\right)f_p, \qquad 0 \leqslant f_p \leqslant 1,$$

$$p_{\mathrm{w_t}} = p_{\mathrm{w_b}} - \left(p_{\mathrm{atm}} - p_{\mathrm{w_b}}\right)f_p, \qquad -1 \leqslant f_p \leqslant 0.$$

♣ Horizontal 1×3: central pipe held at fully functional pressure p_{w}, with the other two cells operated in the same way as in horizontal 1×2.

♣ Vertical 3×1: central pipe held at fully functional pressure p_{w}, with the other two cells operated in the same way as in vertical 2×1.

♣ 2×2: alteration of suction done in the left pair of wells due to horizontal symmetry; operated identically to vertical 2×1. Horizontal pair with the non-dominant pipe work in the same way as in horizontal 1×2.

♣ The FlexPDE solver[2] was used to obtain a finite element solution with an unstructured triangular, dynamically refined mesh with a prescribed relative error in p (non-dimensionalised by p_{atm}) of 10^{-7}. The flux integrals for all configurations were evaluated using the solver's integration functionality.

Equations

♣ Darcy's law[5] (steady state fluid flow through a porous medium):

$$\mathbf{u} = -\frac{k}{\mu}\left(\nabla p - \rho\mathbf{g}\right),$$

$\mathbf{u}$ – velocity vector, k – effective permeability value, ∇p – pressure gradient (non-constant), μ – gas viscosity (computed based on molar fractions[1]), ρ – fluid density, $\mathbf{g} = (0,-g,0)^{\mathrm{T}}$ – gravity vector.

♣ Mass conservation[3] (steady state):

$$\nabla\cdot(\rho\mathbf{u}) = C,$$

with the generation rate C non-zero in the waste layer only.

♣ Ideal gas equation:

$$p = \frac{\rho}{RT},$$

R – gas constant, T – effective temperature.

The velocity vector $\mathbf{u}$ can be written as $\mathbf{u} = (u,v)$, where u and v are the horizontal and vertical components respectively.

The flux across the centre line of the domain induced by asymmetry, for horizontal and vertical configurations (figures 1 and 2), is defined as:

$$F_{1\times 2} = \int_{-\ell_y}^{\ell_y} \rho u\,\mathrm{d}y, \qquad F_{2\times 1} = \int_{-\ell_x}^{\ell_x} \rho v\,\mathrm{d}x.$$

Horizontal 1×2 results

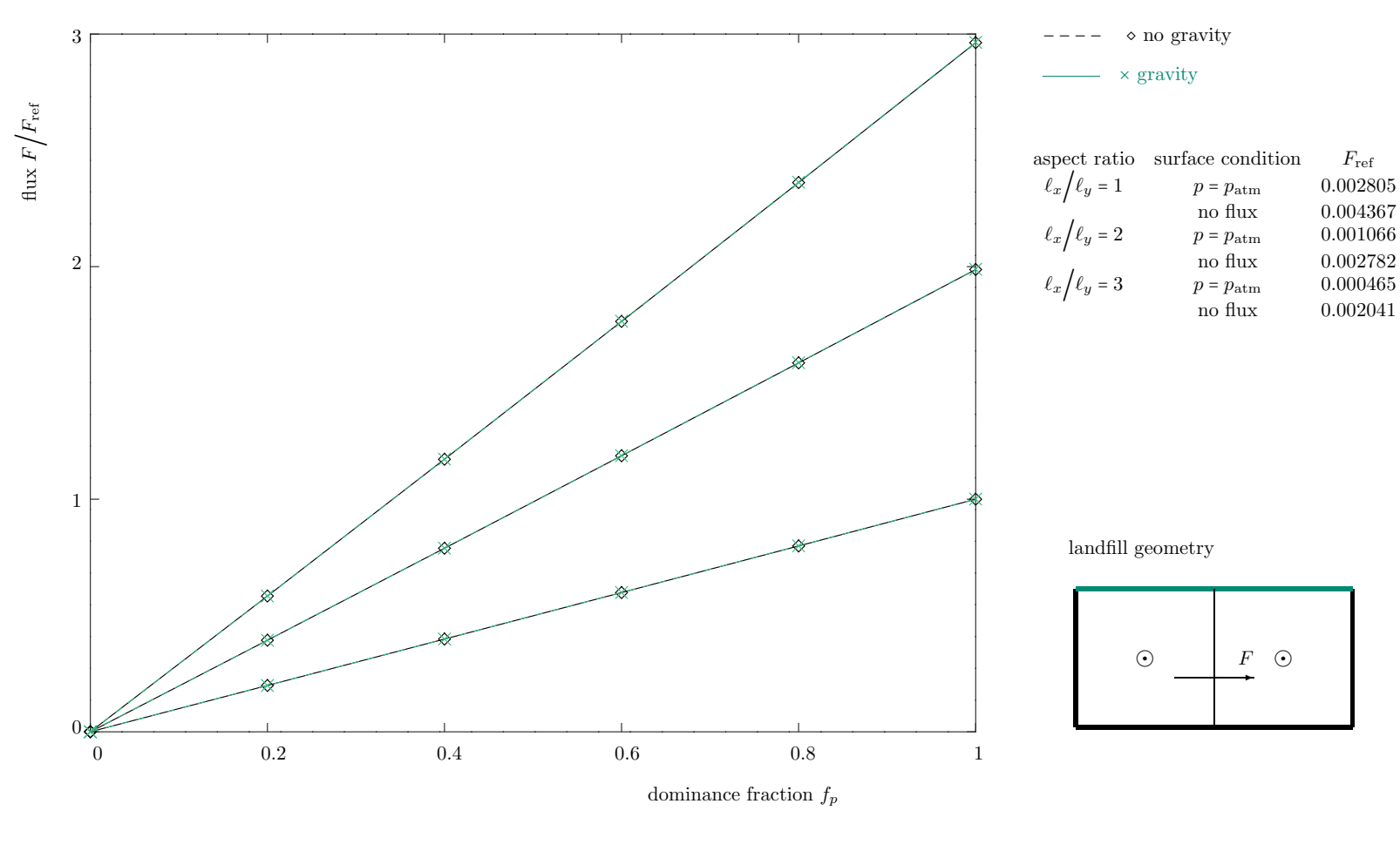

Figure 3 Centre line flux for suction values $p_{\mathrm{w}} = \{-1.25,\ -2.5,\ -3.75\}$kPa (respectively increasing slope) versus pipe dominance fraction f_p. F_{ref} is the value for the case of one well entirely dysfunctional ($f_p = 1$), the other maintaining pressure $p_{\mathrm{w}} = -1.25$kPa, and gravity neglected. Respective F_{ref} for aspect ratios $1 \leqslant \ell_x\big/\ell_y \leqslant 3$ and surface boundary conditions are given.

♣ Figure 3 shows the flux across the centre line for a range of suction values p_{w}. There is a marked linearity throughout the range of f_p, but also in p_{w}. The range $-3.75 \leqslant p_{\mathrm{w}} \leqslant -1.25$ spans the viable working points of medium size landfills.

♣ The range of suction values can also be interpreted as well pressure at sections farther upstream from the outlet, so that these results are valid for any cross-section in the well.

♣ The flow field with a constant pressure condition on the surface differs significantly from the one with a sealed surface. Without a cover the waste matrix shows a moderate to high permeability and low resistance, where the sealed boundary has zero permeability and infinite resistance.

♣ Nonetheless, the results for both types of boundary conditions, when normalised by the respective value of F_{ref}, give identical graphs for the horizontal 1×2 configuration, i.e. are self-similar.

♣ The impact of gravity was minimal as expected for this configuration.

♣ The flux graphs for aspect ratios $\ell_x\big/\ell_y = 2$ and $\ell_x\big/\ell_y = 3$ normalised by their respective F_{ref} are indistinguishable from those corresponding to $\ell_x\big/\ell_y = 1$. This invariance of an integrated quantity is stunning, considering the complexity of the underlying flow field. For instance, the dependence $F_{\mathrm{ref}}\left(\ell_x\big/\ell_y\right)$ for either boundary condition is not linear.

Vertical 2×1 results

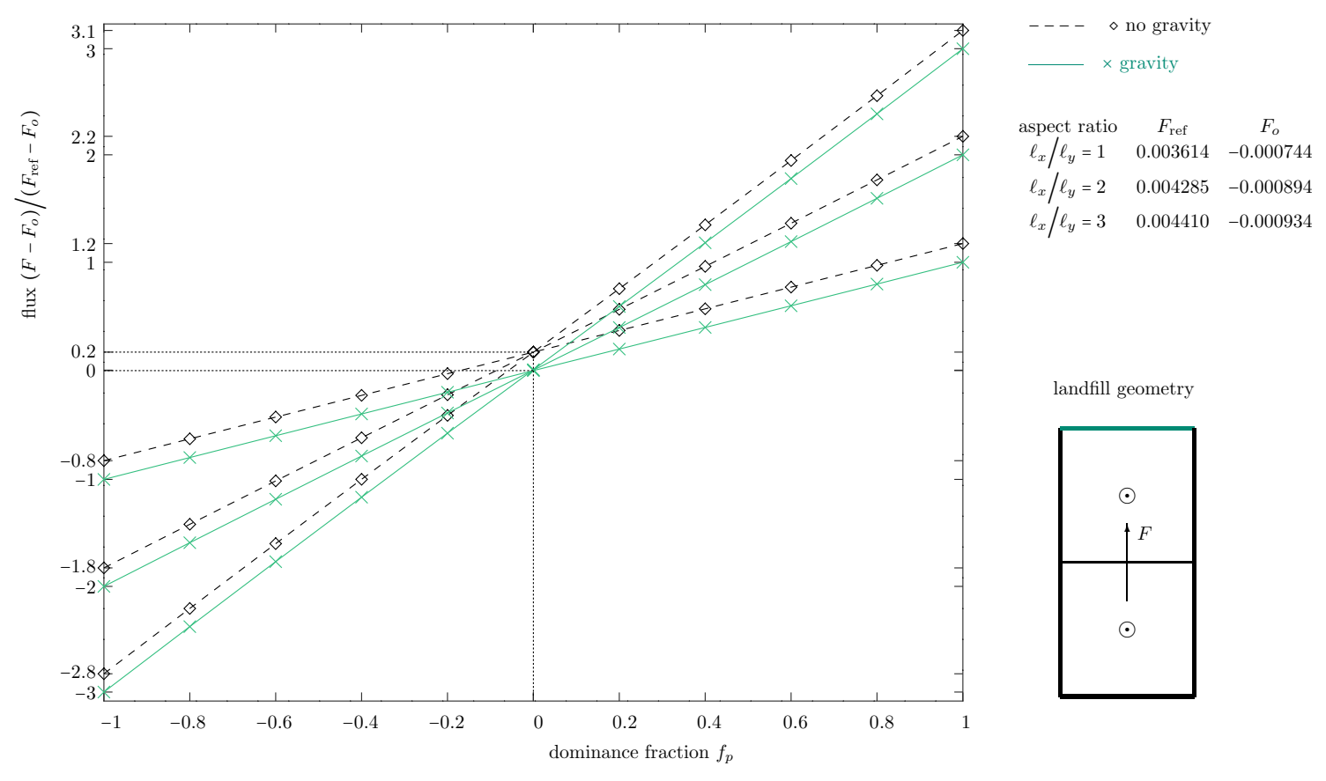

Figure 4 Centre line flux for suction values $p_{\mathrm{w}} = \{-1.25,\ -2.5,\ -3.75\}$kPa (respectively increasing slope) versus pipe dominance fraction f_p with a sealed surface. F_{ref} is the value for the case of bottom well entirely dysfunctional ($f_p = 1$), top well maintaining pressure $p_{\mathrm{w}} = -1.25$kPa, and gravity accounted for. F_o is the value for equally dominant wells ($f_p = 0$). Respective F_{ref} and F_o values for aspect ratios $1 \leqslant \ell_x\big/\ell_y \leqslant 3$ are given.

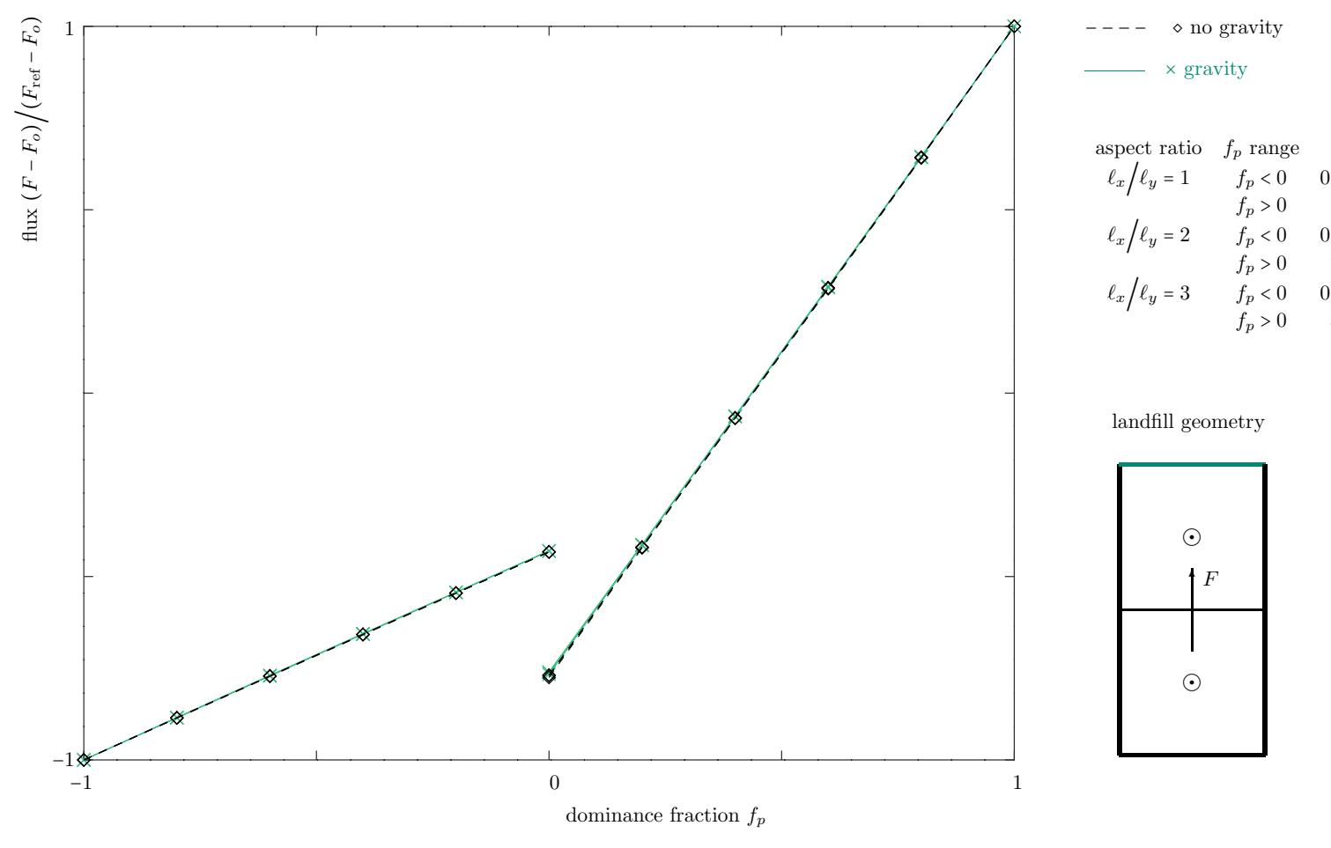

Figure 5 Self-similarity of all flux data with top surface held at constant pressure. The marked values on the ordinate are correct by definition, but the lines are not to scale; for exact proportions see line equations for different aspect ratios given to precision of two significant figures.

♣ Asymmetry is induced both by gravity and different surface conditions. Separate similarity laws were sought for the two possible surface conditions. Regardless of whether gravity is accounted for or neglected, the flux function is linear in f_p. The normalisation value used corresponds to $f_p = 1$, i.e. a dysfunctional bottom well with the top well held at a suction value of $p_{\mathrm{w}} = -1.25$kPa, with gravity taken into account. Linearity in suction strength is evident when comparing the values given by the lines for the three suction values for a fixed f_p.

♣ For intermediate values $-1 < f_p < 1$ the relative error due to the omission of gravity was found to be possibly quite large, as the difference in the flux values was nearly constant at approximately 0.2 for all values of f_p. As such, gravity cannot be ignored in vertical configurations.

♣ The quantitative prediction in figure 4 holds for all aspect ratios tested for the sealed surface configuration, implying a similarity law analogous to the horizontal configuration, a remarkable result with gravity involved.

♣ When the top surface is open to the atmosphere, the dependence of the centre line flux F on suction strength as well as aspect ratio becomes non-linear. The function $F(f_p)$ is linear for the separate regions $f_p \gtrless 0$, but not overall. There exists a critical dominance fraction value $f_{p_{\mathrm{cr}}} > 0$, where F is independent of suction strength. Gravity impacts the flux value at $f_{p_{\mathrm{cr}}}$, but not $f_{p_{\mathrm{cr}}}$ itself. Using ($f_{p_{\mathrm{cr}}}$ as a centring point enables a similarity law with respect to suction strength for $f_p > 0$. $f_{p_{\mathrm{cr}}}$ grows with landfill aspect ratio, but at the higher aspect ratios this growth slows down as the well's radius of influence is reached and more landfill gas escapes collection.

♣ When the flux F_o at $f_{p_{\mathrm{cr}}}$ is used to centre the lines and the flux F_{ref} at $|f_p| = 1$ to scale them, all lines in figures 3 and 4 collapse onto a single line connecting the points $(-1,-1)$ and $(1,1)$, i.e. basically

$$(F - F_o)\big/(F_{\mathrm{ref}} - F_o) = f_p$$

for any suction strength, aspect ratio and boundary condition.

♣ When the surface is not sealed, this procedure must be performed separately for $f_p \gtrless 0$, resulting in figure 5. This shows that the only dependence is on the aspect ratio. The coefficients of the equations are virtually linear in aspect ratio and thus an additional tier of normalisation would result in a single formula (still split for $f_p \gtrless 0$).

Practical Application

The linear behavior of the normal flux between adjacent cells is conducive of building empirical models, when practitioners have no access to advanced numerical computation. The vertical 2×1 configuration with a sealed surface and the horizontal 1×2 configuration with either condition on the surface are only weakly asymmetric and possess full similarity throughout the parameter space. The value of F_o, where the wells are operating at equal suction, as well as the value F_{ref} at any non-zero value f_p can be used in order to construct one straight line $F(f_p)$. Implementing the similarity laws, this function can be used for prediction of the flux (difference in production between the two wells) under any suction value or suction asymmetry condition.

Situations where these predictions may prove helpful include:

♣ replacement of failing vacuum blowers with stronger or weaker models;

♣ possible damage to the bottom pipe in the 2×1 configuration;

♣ given an asymmetric suction application and a desired production, it is possible to compute the required blower strength;

♣ if the cell is expanded to a greater aspect ratio or an additional well is installed, diminishing the aspect ratio, designating a reference point on the formerly obtained line and correcting by the new flux at the same operational point will provide all required predictions for the new configuration.

For the vertical 2×1 configuration with a partly permeable surface, the qualitative behaviour remains linear, but $f_{p_{\mathrm{cr}}} \neq 0$ and it is necessary to split the function for $f_p \gtrless 0$. The vacuum blower should be tuned between two or three values and production of the two wells for the values $f_p = -1, 0, 1$ recorded. Constructing the respective lines and calculating their intersection point separately for $f_p \gtrless 0$ yields $f_{p_{\mathrm{cr}}}$. The flux at that point is F_o. F_{ref} is the value at some designated point $f_p > 0$.

Horizontal 1×3 results

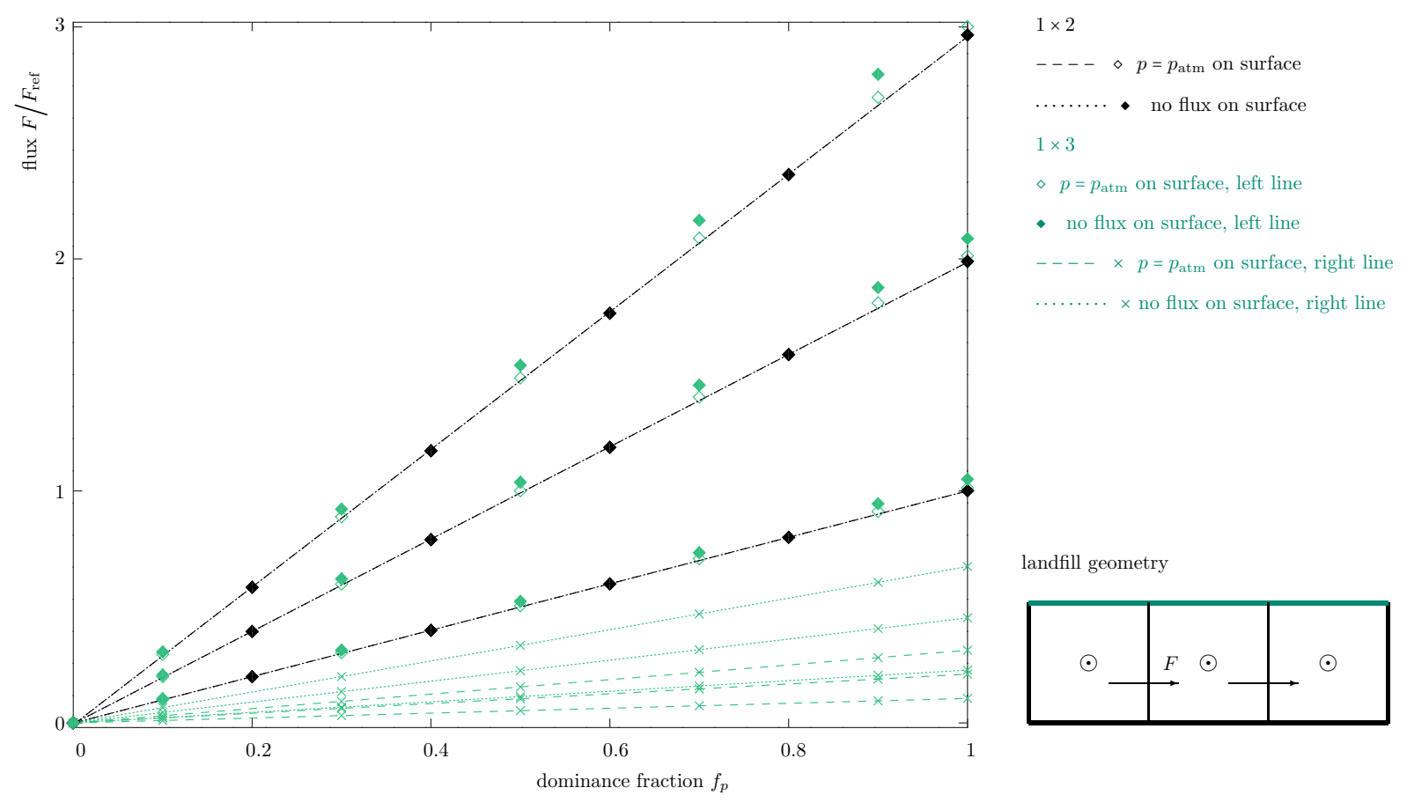

Figure 6 Line flux for suction values $p_{\mathrm{w}} = \{-1.25,\ -2.5,\ -3.75\}$kPa (respectively increasing slope) versus pipe dominance fraction f_p with no gravity. F_{ref} is the value for the case of the left well entirely dysfunctional ($f_p = 1$), the other two maintaining pressure $p_{\mathrm{w}} = -1.25$kPa. Aspect ratio $\ell_x\big/\ell_y = 1$. F_{ref} values are given in figure 3.

Most landfills in real life do not have just one or two cells. For the 1×3 horizontal configuration the following comparison was performed. The conditions in the central and right cells were kept fixed. The suction strength of the left cell was diminished as explained in the methodology section. Figure 6 shows straight lines fitted through the points of the 1×2 configuration, whereupon the points of the 1×3 configuration were added to the graph. The visual agreement implies that essentially the flux between the left and central cells depends on the relative dominance between those two cells and is independent of the state of the right cell. The highest relative error was 5% for a sealed surface. Therefore the horizontal 1×2 analysis can be performed for configurations of multiple cells upon division into suitable units of two cells. The lesser suction at the left cell produced a small flux (an order of magnitude less) across the contiguity line between the central and right cells. This residual flux behaves according to the self-similarity laws discussed heretofore, however no coherent quantitative relation was found to connect it with the main flux.

Vertical 3×1 results

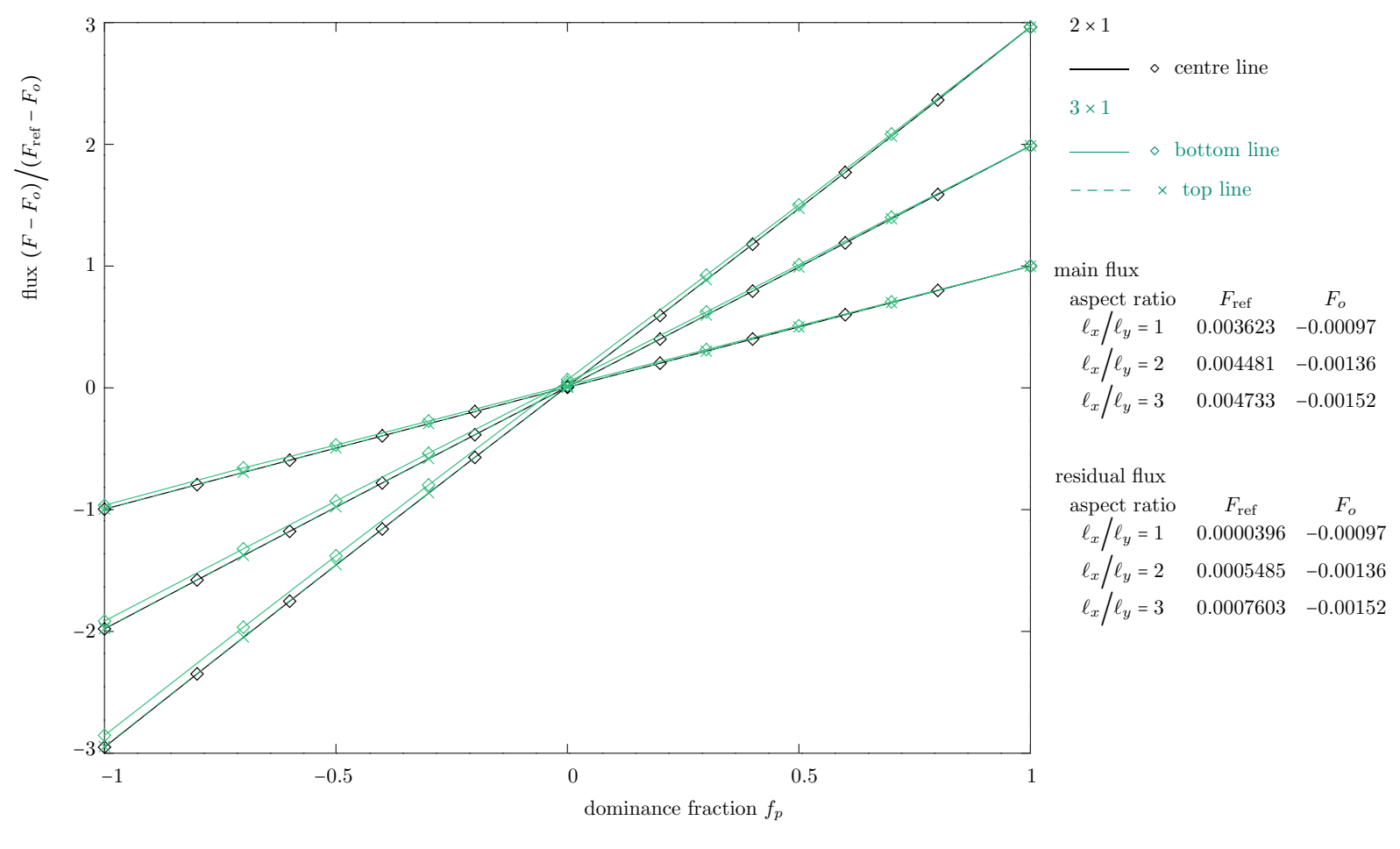

Figure 7 Line flux (non-dimensional) for suction values $p_{\mathrm{w}} = \{-1.25,\ -2.5,\ -3.75\}$kPa (respectively increasing slope) versus pipe dominance fraction f_p with a sealed surface, gravity accounted for and central well maintaining pressure $p_{\mathrm{w}} = -1.25$kPa. F_{ref} is the value for the case of bottom well entirely dysfunctional ($f_p = 1$), top well maintaining pressure $p_{\mathrm{w}} = -1.25$kPa. F_o is the value for equally dominant wells ($f_p = 0$). Respective F_{ref} and F_o values for aspect ratios $1 \leqslant \ell_x\big/\ell_y \leqslant 3$ are given.

The analysis of the vertical 3×1 configuration is a similar extension of the 2×1 counterpart. The central cell was kept fixed and the suction strength of the top and bottom cells were manipulated as explained in the methodology section. Gravity was included in all simulations. As with the 1×3 array, there is a strong flux (referred to as main) between the central cell and the compromised cell, and an order of magnitude weaker (residual) flux between the central cell and the other dominant cell. The main flux is well correlated with the central line flux of the 2×1 configuration, however the agreement deteriorates with aspect ratio: the maximal error (at a dominant bottom cell) grows from about 5% for $\ell_x\big/\ell_y = 1$ to 20% at $\ell_x\big/\ell_y = 3$. Both the main and residual fluxes are linear throughout the range $-1 \leqslant f_p \leqslant 1$. From the results of the 2×1 configuration with a permeable surface, no self-similarity is expected as the three independent sources of asymmetry (boundary condition, suction strength and gravity) render the flow field highly asymmetric. Both the main and residual fluxes exhibited qualitative behaviour as in the 2×1 configuration.

2×2 results

When the suction is diminished in the bottom left well, some of the gas generated in that cell travels into the other three cells. Some is collected by the bottom right well, whilst the top left well collects a slightly smaller amount due to gravity. Most of the laterally moving mass in the bottom pair of cells is absorbed by the right well. However, a residual flux does continue vertically into the top right cell. When comparing the flux across the horizontal centre line of 2×2 and 2×1 arrays, only the flux across the left half of the line should be taken in the former case, to avoid including any superfluous mass counted both as a horizontal flux between the two bottom cells, and as a vertical flux between the right pair of cells.

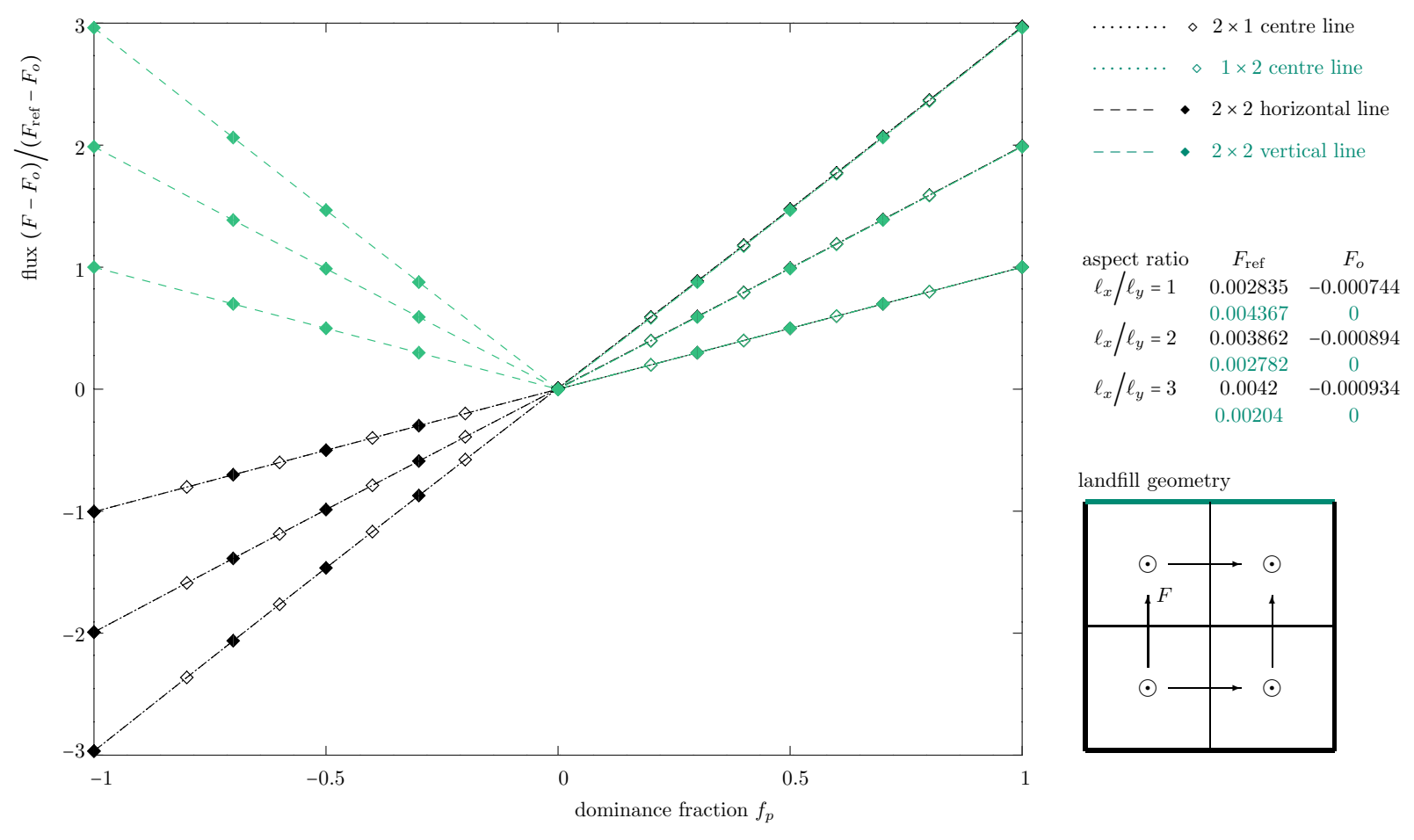

Figure 8 Line flux (non-dimensional) for suction values $p_{\mathrm{w}} = \{-1.25,\ -2.5,\ -3.75\}$kPa (respectively increasing slope) versus pipe dominance fraction f_p with a sealed surface, gravity accounted for and right wells maintaining pressure $p_{\mathrm{w}} = -1.25$kPa. Respective F_{ref} and F_o values for aspect ratios $1 \leqslant \ell_x\big/\ell_y \leqslant 3$ are listed.

The flux between the left pair of cells is expected to be slightly less than that in the 2×1 configuration due to the possibility of lateral escape to the right half of the landfill. The quantitative agreement between the vertical flux in the left half of the landfill and the 2×1 counterpart improves with aspect ratio, as the lateral escape is minimised (black dashed and dotted lines). All of the above fluxes obey similiarity laws and exhibit linearity in suction strength, as confirmed in figure 8. With a partly permeable surface, qualitatively the same properties were recovered as for 1×3 and 3×1 configurations (linearity and $f_{p_{\mathrm{cr}}}$). Hence all fluxes can be represented as in figure 5.

Conclusions

♣ The landfill gas flow field was studied in a realistic geometry in conjunction with asymmetric operational conditions. Asymmetry due to a partly permeable surface, gravity, unequal suction strength at multiple collection wells and combinations thereof was considered. It was shown that integrated quantities such as normal flux across a contiguity plane between two adjacent cells might exhibit a markedly simple behaviour over a stunningly wide range of parameters.

♣ What makes the normal flux a unique quantity in landfill gas flow field modelling is its immediate accessibility in the field. It is just about the only parameter that can be obtained with little effort and high certainty: it simply equals the difference in the mass of gas collected by adjacent wells as measured at the wellheads. Thus, this quantity can be tracked over time and all associated models adjusted accordingly with relative ease.

♣ The main factor affecting the behaviour of the flux between cells is the number of asymmetry sources. With small to medium asymmetry the flux manifests bi-linearity with respect to the relative suction strength as well as absolute suction strength, and the dependence on both is self-similar in aspect ratio.

♣ In the 1×2 configuration the main source of asymmetry is a partly permeable cover, with a weaker gravity effect. With a sealed cover, the only source of asymmetry is gravity, and both cases would fall into the small to medium asymmetry category. When more cells are positioned in a row, as in the 1×3 configuration, the geometric asymmetry of suction points is added, causing a small deviation from the perfect two-cell self-similarity. In practice it would be reasonable to focus the modelling on any two adjacent cells out of a row, providing that the weaker wells are separated by at least two fully functional ones.

♣ In vertically stacked cells the gravity becomes an important physical factor, so that with an impervious cover the asymmetry is still medium, however with a partly permeable surface the asymmetry is high enough for the overall linearity in the dominance fraction f_p to break down. However, the linearity is retained separately for the ranges $f_p \gtrless 0$. There exists a critical value $f_{p_{\mathrm{cr}}}$, where the flux is independent of suction strength. The linearity in suction strength is impaired slightly, however the practical use of self-similarity should not be impeded in any way. The analysis is valid for configurations of three cells and more with the same stipulation as for horizontal arrays.

♣ The sealed and partly permeable surface boundary conditions delimit the moderate to extremely low range of feasible operating permeabilities. The existence of exact similarity laws for an impervious cover and a minor deterioration of bi-linearity as the cover resistance is diminished, allow to conjecture that using these models under most operating conditions would entail only a small error. The landfill engineers would be able to quantify the error by constructing additional lines $F(f_p)$ as required.

♣ The analysis is valid for any cell aspect ratio as long as adjacent cells are within each other's radii of influence. For any given suction value there exists a cell so oblate $\left(\ell_x\big/\ell_y \gg 1\right)$ that gas generated near its borders is not collected, where these relationships disintegrate.

Summary

The flow field of landfill gas in configurations with multiple suction points was studied for effects of asymmetry. In adjacent horizontal cells the flux across the centre line induced by unequal suction was found to be a linear function of the difference in suction strength as well as of the suction strength itself. The non-dimensionalised flux is invariant with respect to the cell aspect ratio and the surface boundary condition. This self-similarity in conjunction with bi-linearity enables practicable predictions in the field to aid in the operation and modification of one or multiple cells. The linearity and self-similarity hold for a wide range of operational parameters corresponding to a medium sized landfill, but deviations therefrom are observed under highly asymmetric conditions, such as in vertically stacked cells with a partly permeable surface, where gravity becomes an important source of asymmetry. Based on the analysis an adjustable modelling technique is suggested. The presented models are simple to construct, customise, implement and update over the lifetime of a given landfill.

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